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EVALUATION OF A CONICAL-RECEIVER RADIOMETER

ARTHUR E. McNUTT

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GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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ABSTRACT

This report describes two versions of a cone radiometer (an absolute radiometer based on electrical substitution methods), one for operation in a vacuum, the other enclosed in its own small vacuum chamber for operation under ambient conditions. A discussion of the theory of operation of the radiometer includes measurements from standard sources and other sources to illustrate the performance of the cone and describes a series of special tests to determine the validity of the radiometer, including errors and uncertainties associated with it.

PROJECT STATUS

This document is the final report on the evaluation of the characteristics of an absolute conical-receiver radiometer developed at Goddard Space Flight Center. This type of radiometer was used in the August 1967 NASA 711 Solar Measurement Program.

AUTHORIZATION

Test and Evaluation charge number: 322 - 124 - 09-27-01

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INTRODUCTION

The two major sections of this report are an evaluation of the performance of a cone radiometer built for and used in the August 1967 NASA 711 Solar Measurement Program[1], operating under ambient conditions (in that it is maintained in its own environmental control system), and an analysis of the actual as opposed to the ideal performance of the radiometer.

THEORY OF OPERATION

The conical-receiver radiometer operates by electrical substitution. The radiometer consists of fine wire wound tightly into a cone, covered outside with a thin layer of epoxy to hold its shape, and painted inside and out with an absorptive paint (Parson's matte black lacquer). Heating the cone electrically keeps it at a constant temperature. Any change in the radiant flux incident on the cone changes the amount of electrical energy required to maintain the cone at the constant temperature.

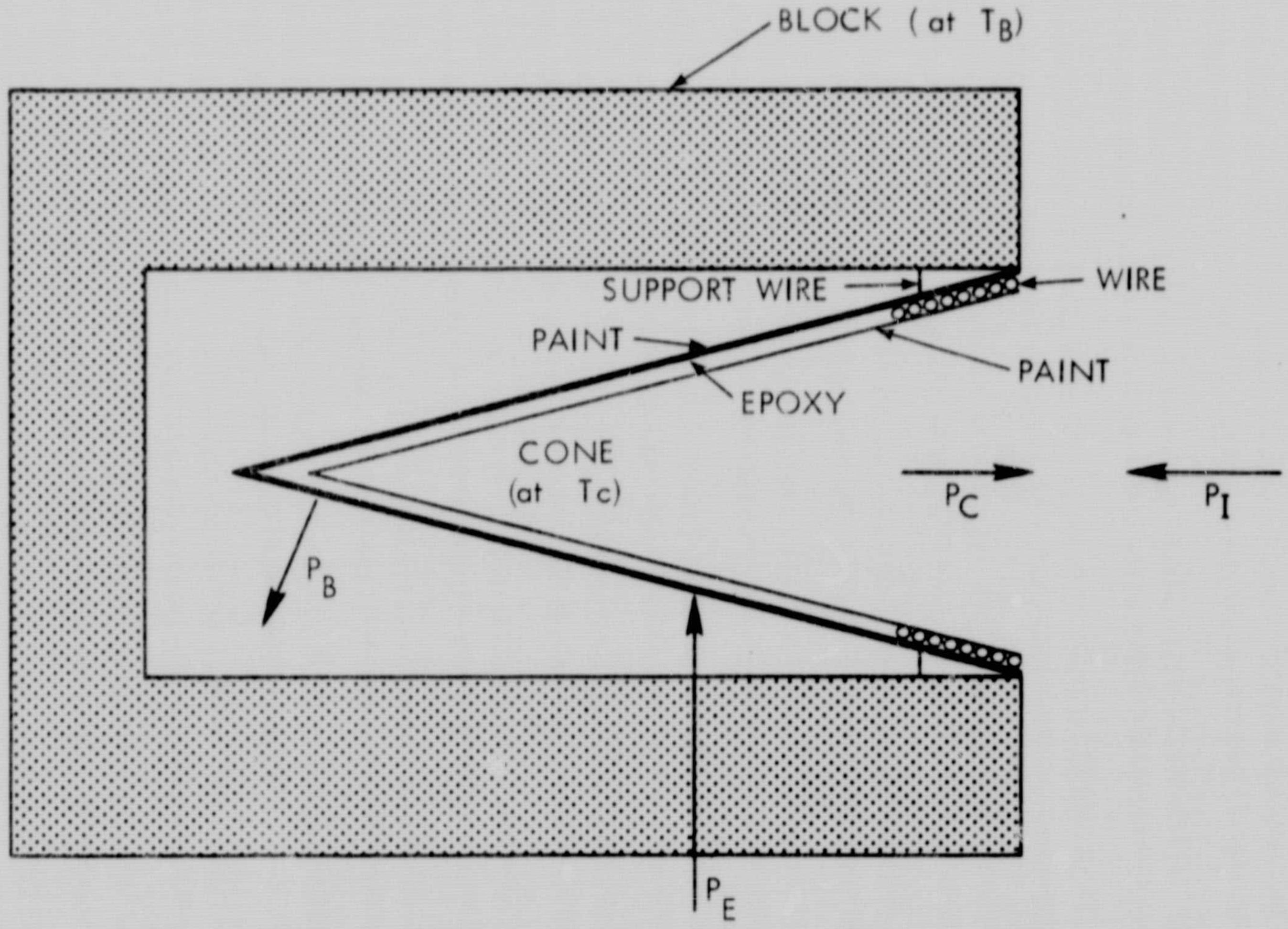
The cone is surrounded, except for its base, by a copper block maintained at a constant temperature; thus, the net energy transfer between the cone and the block is constant. Any change in electrical energy thus reflects a change in the radiant flux incident on the base of the cone only. The cone and block operate in a vacuum chamber at a pressure less than 10^{-3} N/m^2.

Figure 1 shows the cone at a temperature T_C surrounded except for its base by the block at a temperature $T_B < T_C$. Thus, the net power transfer P_B between cone and block is toward the block. P_B consists of a radiative component plus a conductive component contributed by the electrical leads to the cone and the wires supporting the cone. In ambient pressure less than 10^{-3} N/m^2, the gaseous convective and conductive components are negligible; P_B remains constant because T_B and T_C are held constant. P_C, the power leaving the cone through the base, remains constant because T_C is held constant. P_I, the power entering the cone through the base, is the quantity to be measured. P_E, electrical power, maintains the cone at the temperature T_C. Thus:

$$P_E = P_B + P_C - P_I$$

As P_B and P_C are constant for a given T_B and T_C, any change in P_I is equal to the change in P_E. Once having calculated the value of P_{E_0} for $P_I = 0$, any other value for P_E represents an absolute measurement of P_I (i.e., $P_I = P_{E_0} - P_E$).

[1]Matthew P. Thekaekara, ed., The Solar Constant and the Solar Spectrum Measured from a Research Aircraft, NASA TR-R-351, October 1970.



P_B - NET POWER TRANSFER BETWEEN CONE & BLOCK
P_E - ELECTRICAL POWER SUPPLIED TO CONE
P_C - POWER LEAVING CONE THROUGH THE BASE OF THE CONE
P_I - POWER ENTERING CONE THROUGH THE BASE OF THE CONE

Figure 1. Cone Radiometer Cross Section

The temperature of the cone is kept constant by keeping the resistance of the cone wire constant. This is done automatically by a servocontrolled bridge circuit. The cone is said to be nulled when the bridge is balanced and the cone is at the resistance that corresponds to a preset temperature. The electrical power is calculated from the voltage and current through the cone.

In order to operate under room conditions, the cone and its surrounding block must be enclosed in a small vacuum chamber that has a window to admit incident radiation. The surrounding block is maintained at the temperature of liquid nitrogen (Figures 2 and 3).

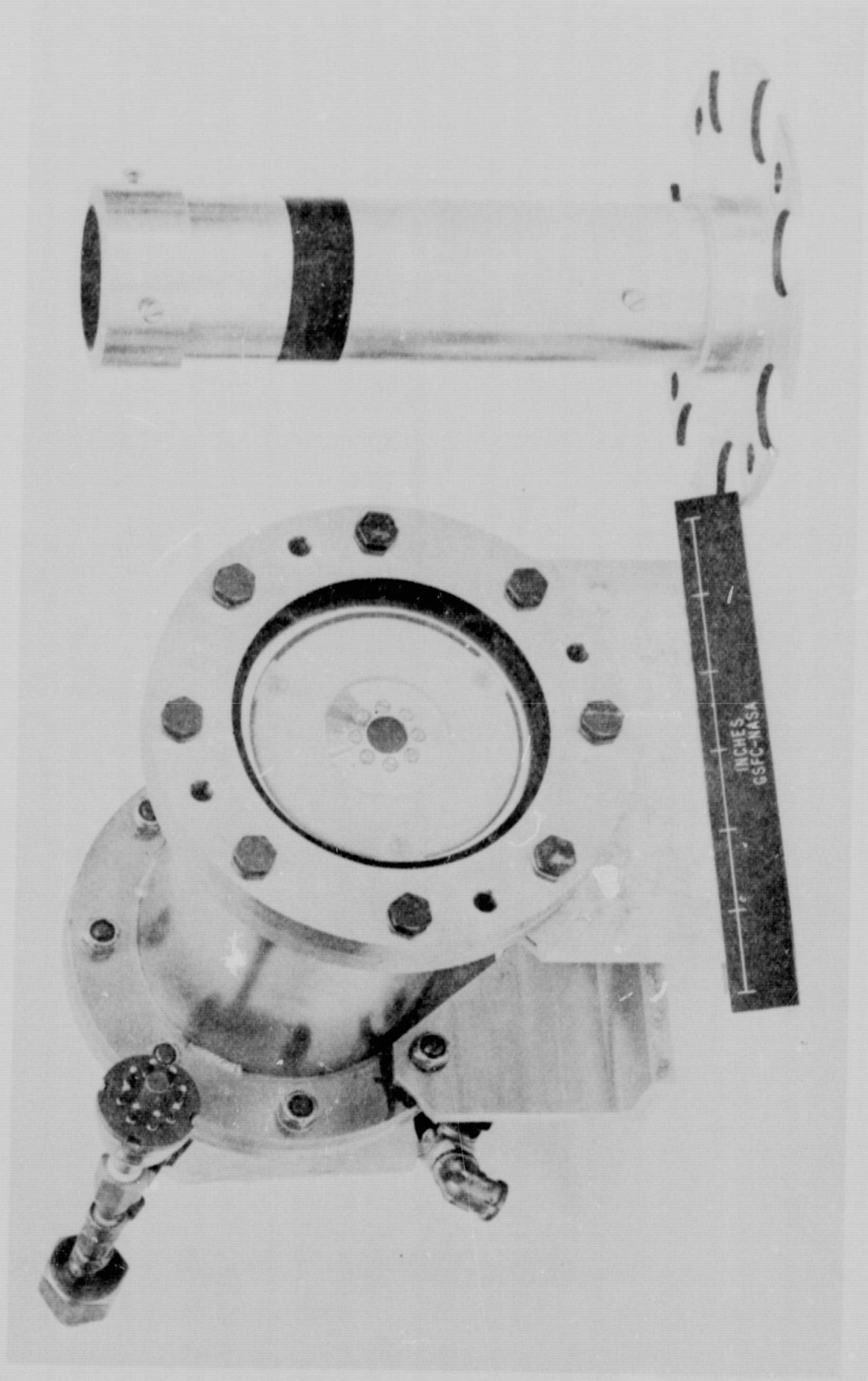


Figure 2. Cone Radiometer

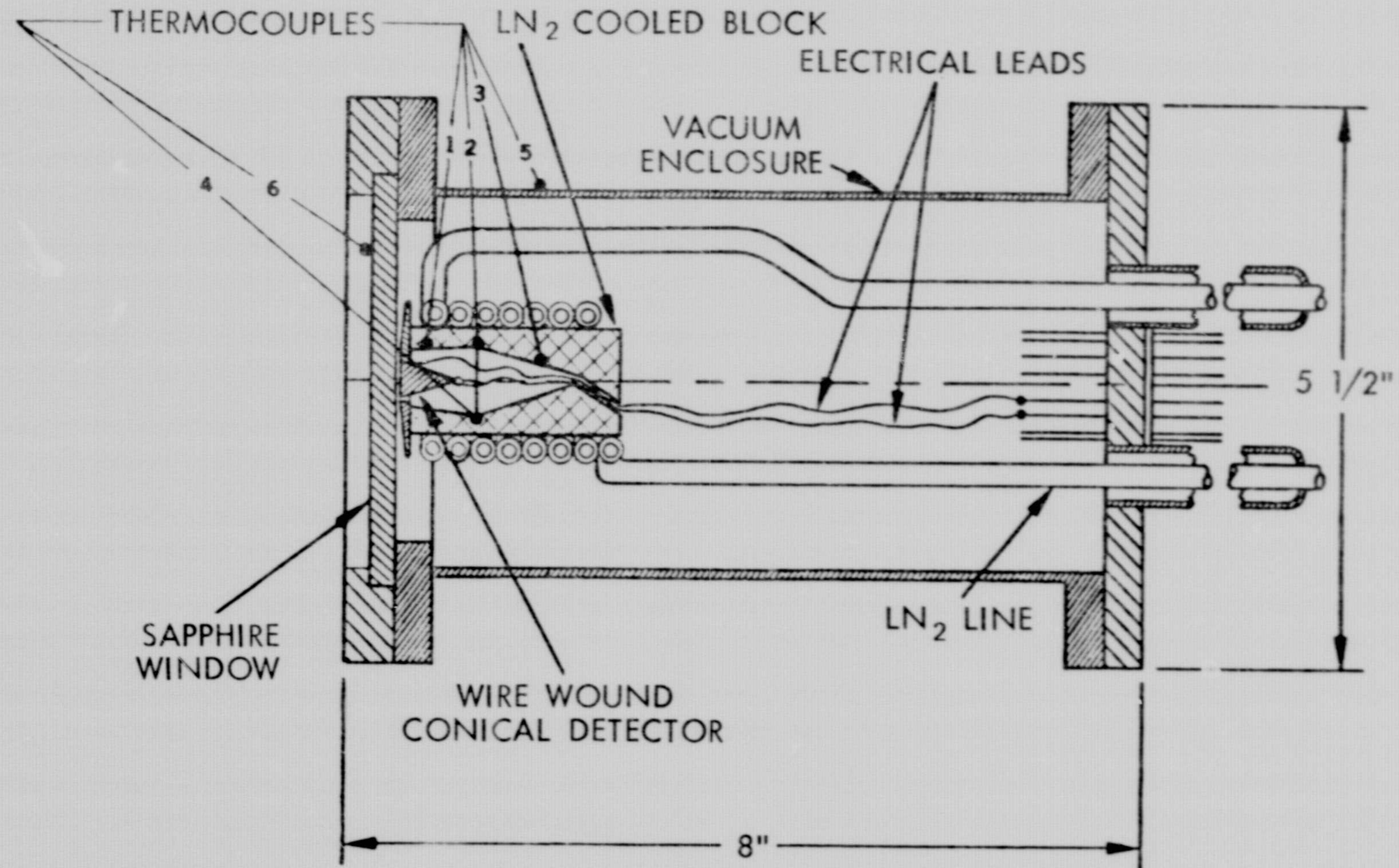


Figure 3. Cross Section of Cone Radiometer

TESTS USING CALIBRATION SOURCE E5K-674

Reflections from Support Ring

A series of tests on the cone radiometer, using the E5K-674 5000-watt projection-lamp calibration source, produced measurements over a period of several weeks under varying conditions. Table 1 lists the twenty measurements made at a

Table 1

Reflection Test on Cone Radiometer Using E5K-674 Lamp at 1 m

Condition	Date	Absorbed Power (mW)	Cone Resistance (ohms)	Chamber Pressure (N/m^2)	Shroud Temperature (°K)	Null Power (mW)
Painted	12/1/67	23.00	210.25	0.51	95	196.65
Painted	12/1/67	23.02	210.24	0.49	96	197.25
Painted	12/1/67	23.06	210.25	0.51	95-96	196.75
Painted	10/20/67	23.10	210.28	0.39	89-96	190.40
Painted	12/5/67	23.11	154.97	0.44		88.45
Painted	12/5/67	23.13	154.97	0.44		88.45
Unpainted	11/22/67	23.37	216.52	0.63		220.40
Unpainted	11/22/67	23.40	206.16	0.63		196.00
Unpainted	11/27/67	23.42	216.53	0.40		198.05
Unpainted	11/27/67	23.42	226.95	0.40		222.75
Unpainted	11/27/67	23.43	210.25	0.40		184.40
Unpainted	11/27/67	23.44	210.25	0.39		183.85
Unpainted	11/27/67	23.48	210.25	0.39		184.00
Unpainted	11/27/67	23.49	206.13	0.39		175.00
Unpainted	10/10/67	23.52	210.01	0.31	90-91	186.50
Unpainted	11/22/67	23.54	210.23	0.63		206.20
Unpainted	10/9/67	23.59	210.02	0.87	93-94	236.30
Unpainted	10/4/67	23.71	210.61	0.69	90-92	225.55
Loose paint	10/23/67	23.76	210.30	0.71	90-91	221.80
Loose paint	10/26/67	23.77	210.28	0.25	91-93	170.60

distance of 1.00 m from the lamp. The cone was in the same physical configuration as during the August 1967 Solar Measurements Program for 11 of the 20 measurements, except that the 0.087-rad acceptance-angle aperture system was removed. The average measured power was 23.46 mW with a scatter of 23.37 to 23.59, a difference of 1.0%.

The ring of metal supporting the cone was highly reflective and had a rounded edge that could reflect light onto the cone; the light-reflection area of the support-ring surface was painted black, using 3M velvet. Nine of the twenty measurements were made with the surface painted; in three of these, loost paint caused erroneous measurements. The six measurements with secure paint averaged 23.07 mW with a scatter ranging between 23.00 and 23.13, a difference of 0.6 percent. The difference between painted and unpainted measurements was 0.38 mW average. The 3M velvet has a reflectance of 0.20 at liquid-nitrogen temperatures; if the reflectance were zero, the difference would have been 0.49 mW. This means that the reflected energy from the ring caused the measured power to be in error, 2.1 percent high.

Comparison with E5K Calibration Value

To compare the value measured by the cone with the lamp calibration, several things must be known. The area of the cone was measured from a shadowgraph tracing using a planimeter and was also calculated from a measurement of several diameters; the two methods yielded the same answer with 0.5 percent, $A = 0.7317\ cm^2$ Considering the cone shape and absorptivity of Parsons' matte black lacquer, a value for the effective absorptive area was arrived at, $\alpha A = 0.7228$. The sapphire window transmission was calculated theoretically, measured on the Beckman DK-2 and IR-9 spectrophotometers, and was also measured using an Eppley thermopile. Values range from 80 to 90 percent transmission, depending mainly on the light source. A value of 90 percent (based on the measurements with the Eppley thermopile) was chosen for the E5K-674 lamp. The incident irradiance, I, is given by:

$$I = \frac{P_I}{\alpha AT}$$

where

$$P_I = P_{E_o} - P_E$$

P_I = measured absorbed power

αA = effective absorptive area

T = window transmission

The measured power for E5K-674 at 100 cm is 23.46 mW, minus the 0.49 mW correction for the light reflected from the ring or 22.97 mW. 22.97/0.7228 x 0.90 = 35.31 mW/cm^2. The lamp calibration value is stated as 33.3 mW/cm^2 at 100 cm. The 6-percent difference could be due to several things; e.g., an uncertainty in the window transmission. Eppley Laboratories designates the E5K-674 lamp as a calibration source and not as a standard lamp, so that the calibration values issued with the lamp may be in error by more than ±2 percent, even though Eppley Laboratories literature states, "The calibration accuracy is believed to be better than ±2% . . ."

Choice of Operating Resistance

The resistance of the cone was not the same in all twenty measurements made at 1.00 m with the E5K-674 lamp: values ranged from 206.16 to 226.95 ohms for the eleven unpainted-ring measurements. The six measurements made with the ring painted yielded values from 154.97 to 210.28 ohms. In no case was there any correlation between resistance and the measured value of the power: resistance is proportional to the temperature of the cone. Therefore, within the temperature limits investigated, the measured value of the absorbed power is independent of the temperature at which the cone is operated.

Other Parameters

The measured value of incident power also appeared to be independent of the particular choice of pressure, the block temperature and the zero-null power (that is, the power required to heat the cone to the desired temperature when not exposed to a radiation source). However, the zero-null power is dependent on the resistance at which the cone is operated, the chamber pressure, and the block temperature. The design of the chamber and the type of pump used caused chamber pressure to run higher than 10^{-3} N/m^2 as desired; gaseous convection and conduction were therefore not negligible. The higher the resistance at which the cone is operated, the larger the zero-null current must be; the higher the pressure at which the cone is operated, the larger the null current must be. The data in Table 1 illustrate these two tendencies. There are, however, limits on the null current: There is a maximum value of current which the servocontrol unit can supply. This, in connection with the pressure and block temperature, determines the maximum resistance at which the cone can be operated. The servocontrol can only supply current to heat the cone, not cool it; therefore, the pressure and block temperature and the source to be measured determine the minimum resistance at which the cone can be operated.

LONGER TERM TESTS USING DATA CENTRAL

Stabilization Time

The zero-null power was found to drift during the time a series of measurements was being made, 0.5 to 3 hours, depending on the number of measurements. In order to study this drift, the cone was operated for an 8-hour period on several days, and Data Central (a central monitoring complex) recorded the following nine parameters every 10 minutes:

- Temperature of the block front
- Temperature of the block center top
- Temperature of the block rear
- Temperature of the block center bottom
- Temperature of the vacuum jacket
- Temperature of the window
- Pressure
- Null current
- Null

Figure 3, a sketch of the radiometer, shows the location of the thermocouples.

Several things were observed: Once the liquid nitrogen (LN_2) flow has been established in the block, its temperature was about 83° K. It rose slowly over a period of 2 to 3 hours, then stabilized at about 91° K, except for the front of the block which stabilized at 95° K. The temperature of the vacuum jacket and the window dropped from 299° K to 295° K in about 2 hours and stabilized there. The pressure stabilized at 0.27 N/m^2 after about 1 hour, and the null current stabilized after about 2.5 hours. In one test, the pressure was increased slowly from 0.27 N/m^2 to 0.33 N/m^2 while the temperatures remained at their stable values. During this time, the current increased 2 percent in order to maintain null, or a 4 percent increase in the power supplied to the cone. Thus, the zero-null current was found to depend on the block temperature and the chamber pressure.

Effect of Incident Radiation on Temperature and Pressure

After waiting 3 hours for the cone to stabilize, a test was made of the effect of incident radiation on the above parameters. A "sun gun" (tungsten-iodine lamp) was used as a light source. During the time of exposure to the light source, the null current decreased as expected; as before, the block temperatures remained stable around 91° K ±1° K. The pressure, however, rose from 0.25 N/m^2 to 0.28 N/m^2, then decreased to 0.25 N/m^2 again when the light was removed. The vacuum jacket and window temperatures rose from 295° K to 302° K during exposure, but returned to 295° K when the light was removed. However, window temperature during exposure was not reliable because the light source could heat the thermocouple directly, and cause the rise in temperature.

Effect of Pressure Rise

Exposure to a light source caused the pressure to rise during these tests. This effect was not always present, but when present can cause an error in the following way. The absorbed power is measured by the difference between the power required to null the cone when unexposed and when exposed to a radiation source. So the resistance of the cone remains constant, this amounts to the difference between the square of the exposed and zero-null currents. The zero-null current is measured at one pressure; if exposure to a source causes a pressure rise, the exposed current will be measured at a higher pressure and be larger than if the pressure remained constant. For typical values (R = 200 ohms, pressure 0.27 N/m^2), a pressure change of 0.03 N/m^2 will cause about a 1.6 percent error in the square of the exposed null current. Assuming a zero-null current of 30 ma, the zero-null power is 180 mW; assuming an exposed null current of 16 ma, the power is 51 mW, but the pressure rise will make this value 1.6 percent too large. The absorbed power is 129 mW if no correction is made: but, if the 51 mW is lowered 1.6 percent, the absorbed power becomes 130 mW. The error in the measurement of absorbed power caused by the pressure rise is thus 0.8 percent.

The drift in the null current, therefore, was the result of changes in pressure and the block temperature. Stability of pressure and temperature was reached in 2 to 3 hours; alternate exposure to a source caused slight pressure fluctuations, so the zero-null current drift couldn't be completely eliminated with this particular design of the cone radiometer.

EFFECTS OF BRIDGE CIRCUIT OFF NULL

Tests using a projection-lamp source showed the effect of an off-null bridge in the servocontrol unit. Null is usually less than ±0.01 mV. The amount off null and the corresponding percentage error in the zero-null power, for a cone resistance of 200 ohms and a zero-null power of 187 mW (typical operating values), are:

Amount Off Null (mV)	Error in Zero Power (%)
± 0.01	±0.00
0.10	0.00

Amount Off Null (mV)	Error in Zero Power (%)
± 0.50	±0.04
1.0	0.08
5.0	0.3
10.0	0.7

Simultaneous measurements of the effect of the null on the measured value of the absorbed power gave the results shown in Table 2 for typical values R = 200 ohms, zero null power = 187 mW, exposed null power = 164 mW, absorbed power 187-164 = 23 mW:

Table 2

Effect of Off-Null on Measurement of Absorbed Power

Amount Off Null	Absorbed Power	% Error
0.00 mV	23.44 mW	0.0
0.5	23.48	0.16
1.0	23.60	0.65
10.0	24.78	5.5
0.00	23.44	0.0
-0.5	23.39	-0.20
-1.0	23.34	-0.41
-10.0	22.12	-5.4

The magnitude of the effect will depend on the resistance at which the cone is operated and the intensity of the radiation being measured. However, a null of ±0.01 mV or less is very satisfactory, and even a null as large as ±0.5 mV is acceptable in most cases.

COMPARISON TO STANDARD SOURCE

Table 3 compares measurements made using a 1000-watt total-irradiance standard projection lamp ETK-6704, issued by Eppley Laboratories with those using E5K-674.

Table 3

Results from Calibration Lamp and Standard Projection Lamp

ETK-6704				
d (m)	P_I (mW)	$\frac{P_I - 0.021 P_I}{0.7228 \times 0.873} =$ I (cone)	I (calibration)	% Difference
2.00	1.119	1.737	1.726	0.6
1.00	4.605	7.145	6.904	3.5
1.00	4.540	7.044	6.904	2.0
0.75	8.093	12.556	12.274	2.3
0.50	18.267	28.341	27.616	2.6
E5K-674				
d (m)	P_I	$\frac{P_I - 0.021 P_I}{0.7228 \times 0.90} =$ I (cone)	I (calibration)	% Difference
1.00*	23.46	35.31	33.3	6.0
0.50	97.99	147.5	141.5	4.2
1.50**	10.15	15.60	14.6	6.8

*Average of 11 measurements
**Ring painted black, so 2.1-percent correction for reflected light was not made

Subtracting 2.1 percent of P_I from P_I will correct for the reflection from the support ring, as these measurements were made before the ring was painted black. The window transmission used for the ETK-6704 was experimentally determined to be 0.873.

COMPARISON TO A BLACKBODY CAVITY

Measurements were made at various distances from an Astro Industries blackbody furnace. The temperature, measured with an L&N optical pyrometer, was 2618°K. Total transmission of the sapphire window of the cone, calculated using a table of the blackbody function for 2600° K and the transmission of the window as a function of wavelength as measured on the DK-2 and IR-9, was 0.8237. Using a theoretical calculated transmission as a function of wavelength for sapphire, the calculated value of total transmission was 0.8152. The value 0.8237 was used. As some uncertainty remained as to the location of the zero distance from the blackbody, and what area should be used as the emitting area of the blackbody, supplementary measurements with an Eppley thermopile were made to determine the values for zero-distance location and emitting area. These values are still in question, and a large degree of uncertainty remains in the results of the cone-radiometer tests using the blackbody. Table 4 shows the test results. As the cone ring was painted, the 2.1 percent correction used previously was not necessary.

Table 4

Results of Cone-Radiometer Tests Using Blackbody

Distance (m)	P_I (mW)	$I = \frac{P_I}{0.7228 \times 0.8237}$ (mW/cm^2)	I (blackbody) (mW/cm^2)	% Difference
1.107	11.95	20.08	20.12	-0.20
0.956	16.01	26.90	26.97	-0.26
0.750	25.96	43.61	43.82	-0.48
0.603	40.61	68.22	67.79	+0.64
0.476	65.47	109.99	108.78	+1.11

The radiometer gave values 0.6 to 6.8 percent too high compared to the ETK standard lamp and the E5K calibration sources; compared to a blackbody, the values ranged from -0.5 to 1.1 percent. This rather large scatter of results could be due to several things:

- One possible cause is stray radiation

  One source, reflection from the support ring, has been found and corrections have been made for it. Another source, light passing between the cone and the support ring and being reflected by the shroud onto the outside surface of the cone, has been shown analytically to be negligible.

- Window transmission

  The sapphire window creates three sources of uncertainty: focus problems, heating effects, and selection of a value for transmission. First, in a noncollimated beam, the sapphire window (which has a finite thickness) tends to make the source appear closer than it actually is; the magnitude of this effect was calculated and corrected for. In a collimated beam, this effect is not present. Second, light is absorbed by the window would tend to heat the window; indications are that the window does heat up slightly. No accurate measurements have been made of the magnitude, but it appears to be negligible.

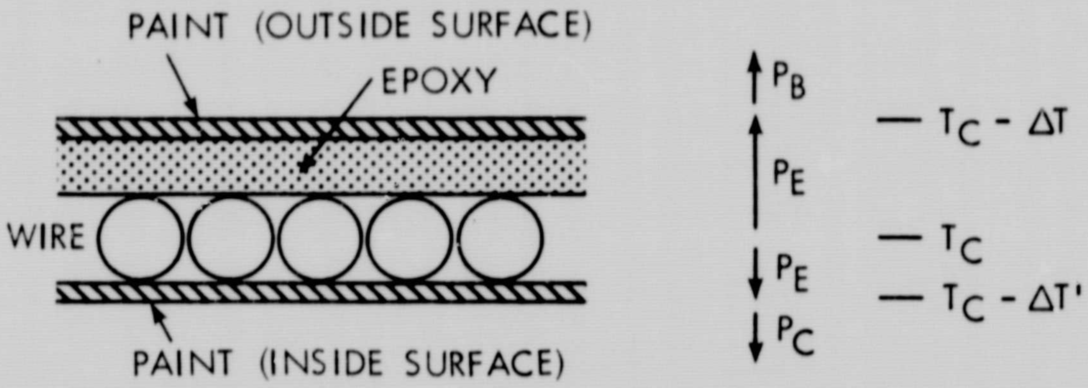
Total transmission is the greatest uncertainty associated with the cone. Values range from 80 to 90 percent, depending on the source and the method of determining transmission. Theoretical and measured values of the transmission as a function of wavelength differ: theoretical values are lower in the region 0.7 to 2.5μ. In the case of 2600 K blackbody spectrum, the difference in the total transmission was 1.0 percent (the difference between 0.8237 and 0.8152). Without knowledge of the spectrum of lamps ETK-6704 and E5K-674, the transmission could not be calculated but had to be measured experimentally. Using an Eppley thermopile with no window, and assuming its paint to be a flat receiver, the transmission of an identical sapphire window was measured for the two lamps, by taking the ratio of the signal with and without the sapphire window in front of the radiometer. For lamp E5K-674, the transmission was 0.90; for lamp ETK-6704, the transmission was 0.865. The transmission of the same window was measured, using the cone for lamp ETK-6704. The result was the transmission was 0.873, 1 percent different from the value measured by the Eppley thermopile. The transmission of the window is now known only to an accuracy of ± 1.0 percent for a given light source, and differs from one source to another.

THE CONE IN A VACUUM CHAMBER

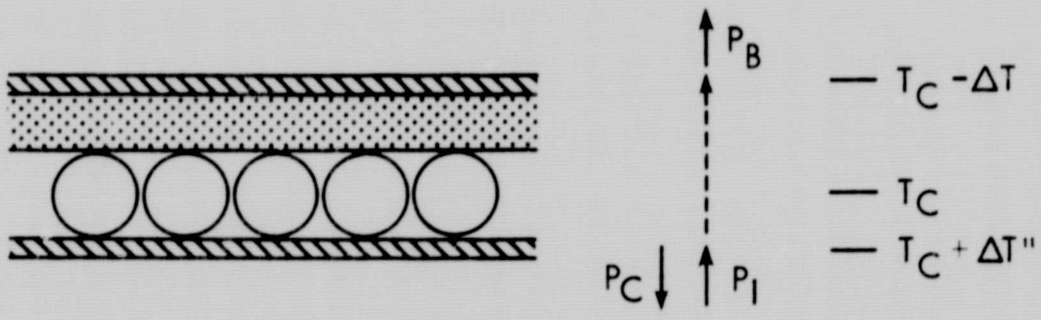
The cone, when used in a vacuum chamber, not in the special configuration used for the Solar Measurements Program, operates on the basis of substituting electrical energy for absorbed radiative energy, which depends on the cone's being in the same physical condition (i.e., having the same temperature distribution) whether electrically or radiatively heated. Two problems that can arise in actual use are:

Determining Cone Temperature

The temperature of the cone is measured by determining the resistance of the cone wire, and this is calculated from the voltage and the current through the cone, which also gives the value for P_E, the electrical power supplied to the cone. By keeping the resistance of the wire constant, the cone temperature is kept constant. The problem is that keeping the resistance constant really only guarantees that the wire is kept at a constant "average" temperature. When $P_I = 0$, all the energy supplied to the cone is electrical and originates in the wire. Figure 4a shows a finite decrease in temperature toward the surface of the cone, in order to conduct heat to the surface to be radiated out as P_C and P_B. Therefore, the surface whose temperature determines P_C and P_B is at a



a) PAINT SURFACE COOLER THAN WIRE AT T_C



b) PAINT ON OUTSIDE SURFACE COOLER THAN WIRE AT T_C
PAINT ON INSIDE SURFACE WARMER THAN WIRE AT T_C

Figure 4. Cross Section of Cone

temperature slightly below T_C. When $P_E = 0$, energy is supplied to the cone by P_I which is absorbed on the inside surface and must be conducted to the outside surface to be emitted as P_B. As Figure 4b shows, the wire remains at T_C and the outside surface will again be at a temperature below T_C, but the inside surface will be at a temperature above T_C. Thus, although T_C, and P_B remain the same in both cases, P_C is greater when $P_E = 0$ than when $P_I = 0$, whereas it should remain the same. As yet no work has been done to measure or calculate the error induced by this effect, which had been assumed negligible compared to other sources of error.

Temperature Distribution Along Cone

Another problem can arise if the surface of the cone is not a uniform temperature. If the temperature is not uniform, the distribution should at least be the same for the two cases considered above ($P_E = 0$ and $P_I = 0$) in order to comply with the requirement that the cone always be in the same physical condition. A series of special cones were constructed to examine the temperature distribution along the surface of the cone and its effect on the power required to heat the cone. One cone consisted of a single wound wire with extra wires tapped at equal distances along the wire, so that the resistance between adjacent taps was the same. Figure 5 illustrates the result, which is that the taps were not spaced equally along the cone surface, although the five area and resistance segments between taps were equal. Measuring the resistance of each segment and comparing it to the total showed whether a given segment was warmer or cooler than the average. A more quantitative way is to use the relation

$$R = R_o (1 + a (T - T_o))$$

where

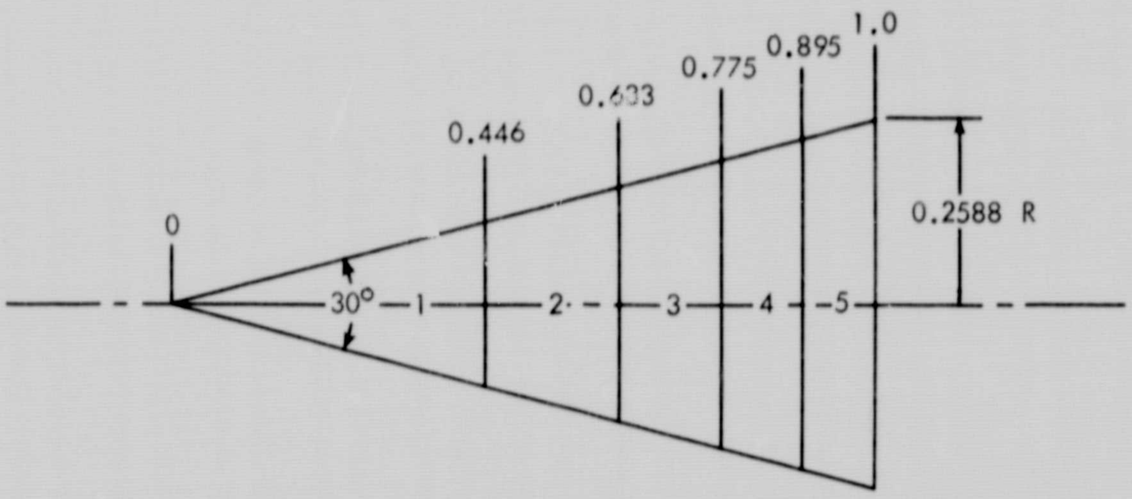
R_o is resistance at temperature $T_o = 273.18°K$

R is resistance at temperature T

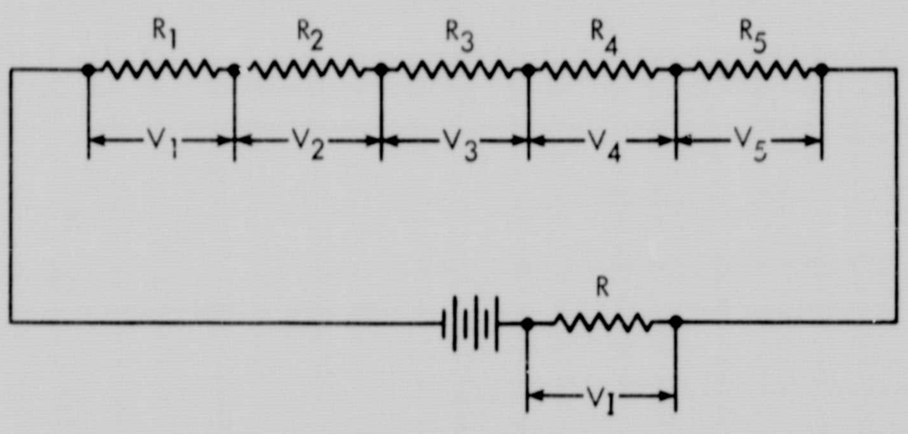
a is temperature resistance coefficient

For each of the five segments and the total length of wire, the resistance R_o was determined for $T_o = 273.18°K$. Knowing the value of a for the particular type of wire, the temperature T could be calculated for any measured value of R. This temperature represents the average temperature of the particular segment. The temperature distribution was examined in four different ways:

- Figure 6 shows the cone and its surrounding block inside a LN_2 cooled can in a vacuum chamber. No electrical or radiant energy, other than that from the chamber walls, was supplied to the cone. The block was allowed to cool down, and the resistance of the cone segments was measured, thus measuring the temperature distribution. The temperature of the block was then increased in steps and the cone-temperature distribution measured for each block temperature. Figure 7 shows the results,



a) RELATIVE DIMENSIONS OF CONE SEGMENTS



$$I = \frac{V_I}{R}$$

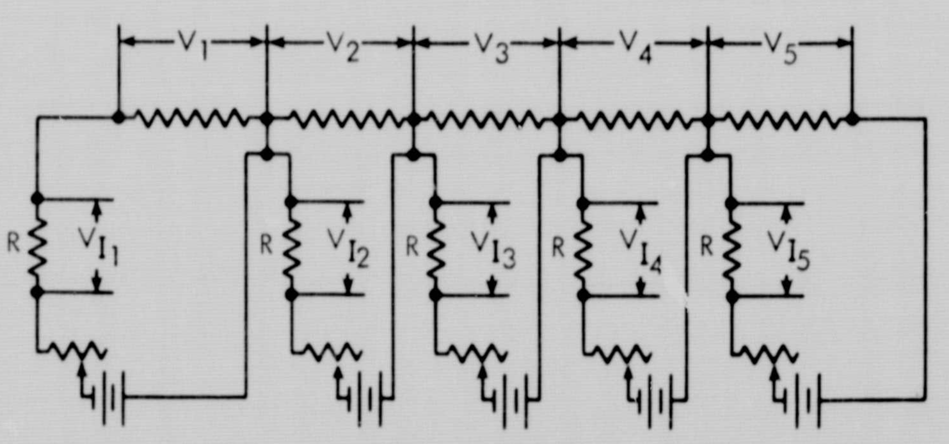
$$R_i = \frac{V_i}{I}$$

$$V = \sum_{i=1}^{5} V_i$$

$$P_E = IV$$

$$R = \sum_{i=1}^{5} R_i$$

b) SERIES CIRCUIT



$$I_i = \frac{V_{I_i}}{R}$$

$$R_i = \frac{V_i}{I_i}$$

$$P_i = I_i V_i$$

$$P_E = \sum_{i=1}^{5} P_i$$

$$R = \sum_{i=1}^{5} R_i$$

c) PARALLEL CIRCUIT

Figure 5. Multiple-Tapped Cone

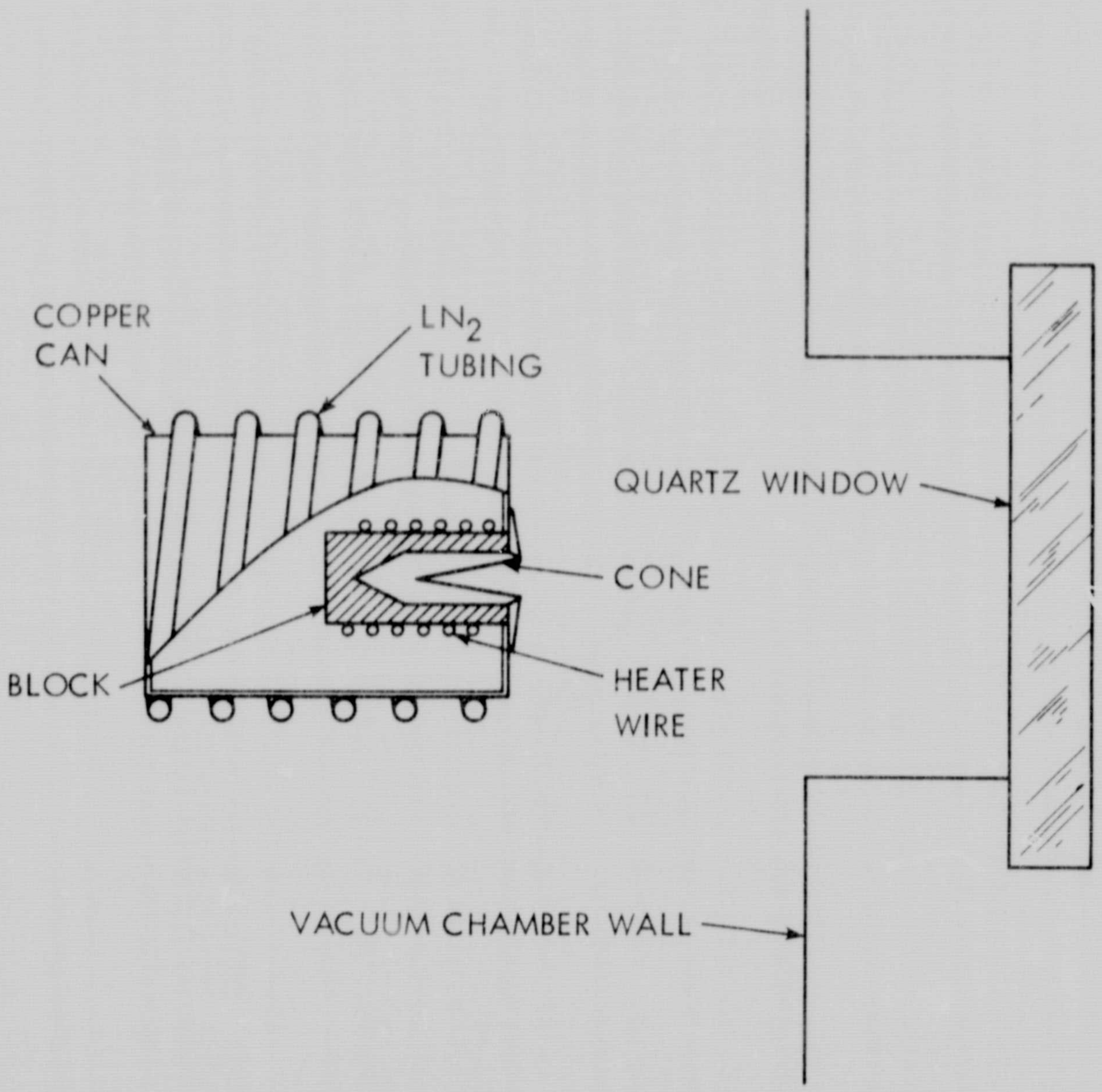
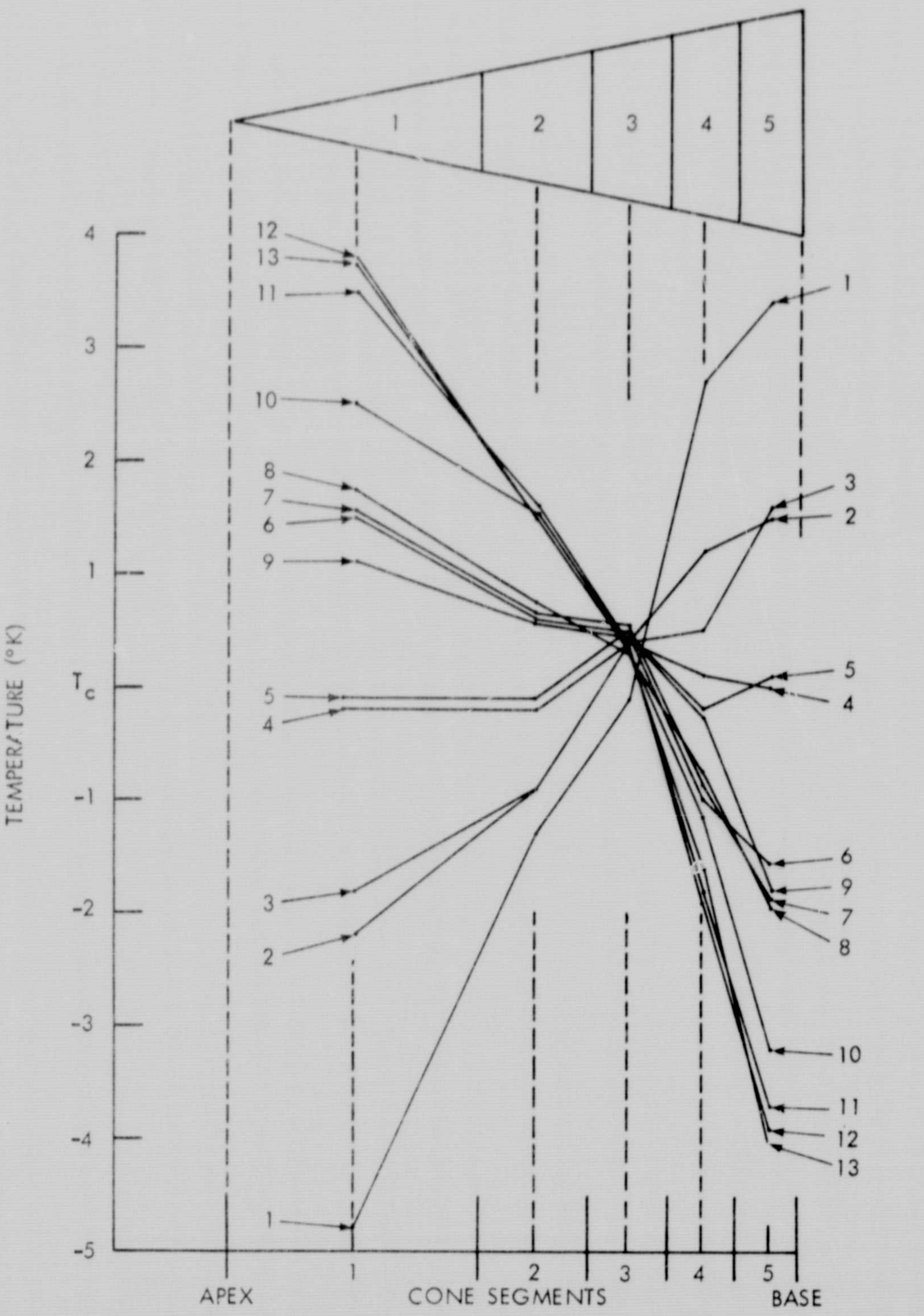


Figure 6. Test Configuration

a clockwise rotation of the distribution about segment three as the temperature is increased. In the graph, the horizontal axis is the position of the segment, its temperature being plotted at the midpoint of the segment. The vertical axis is the number of degrees above or below the temperature of the cone as a whole, T_C calculated from the total cone resistance. The lines connecting data points do not represent a continuous temperature distribution, but serve only to identify the five data points that correspond to a given measurement.

- The block was maintained at a fixed temperature. No radiant energy was supplied to the cone. The cone temperature was increased in steps by electrically heating it, and its temperature distribution measured at the various temperatures. Figure 8 shows the results; again, increasing the

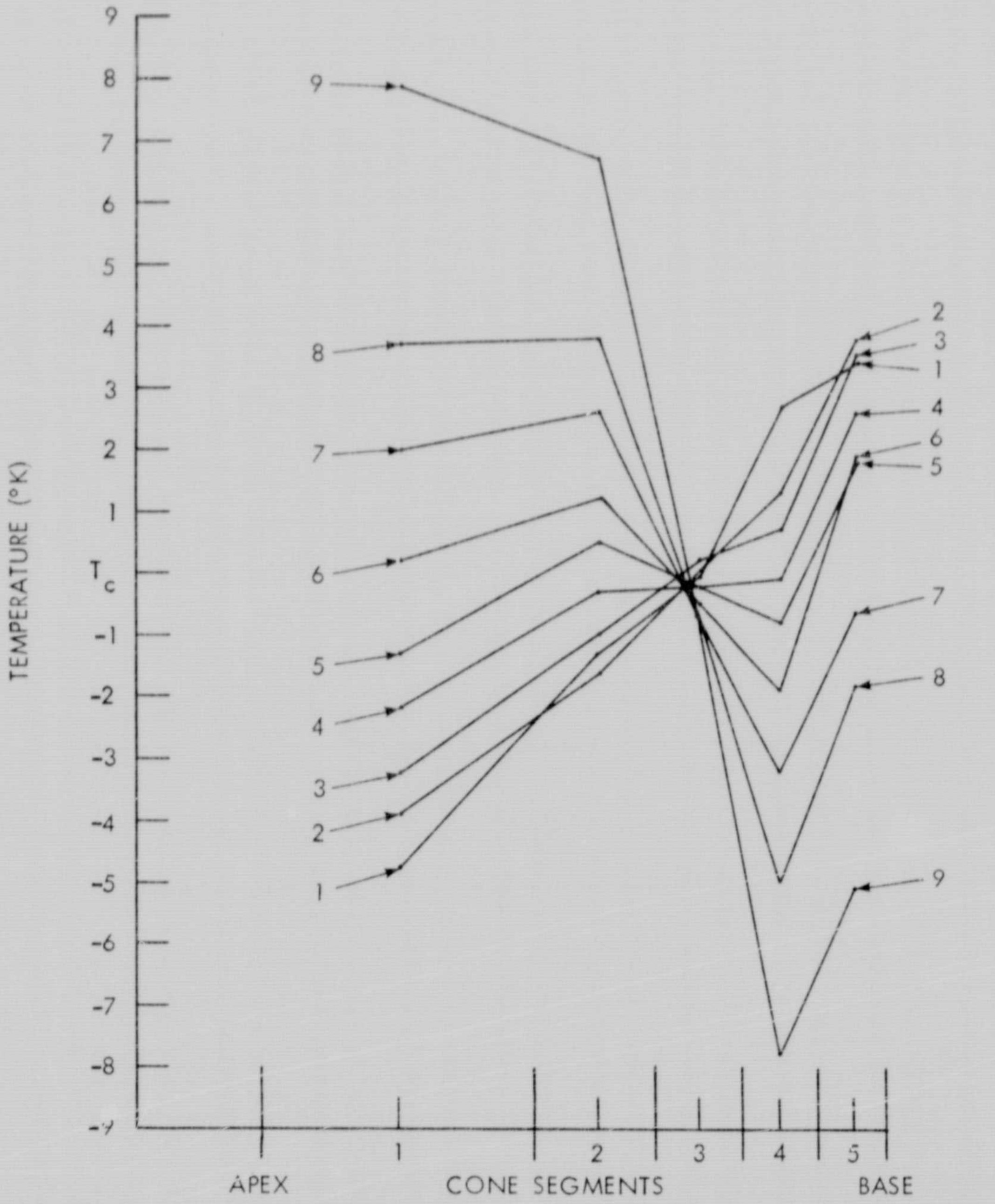


	T (BLOCK)	T_c °K
1	175	185.6
2	223	230.5
3	223	229.2
4	243	247.5
5	243	247.8
6	263	265.4
7	263	264.9
8	271	272.0
9	271	273.0
10	279	279.6
11	289	287.1
12	299	292.1
13	299	291.8

Figure 7. Temperature Distribution as a Function of Block Temperature

cone's temperature caused a clockwise rotation of the distribution about the center of segment three.

- The block was maintained at a fixed temperature. No electrical energy was supplied to the cone. The cone temperature was increased in steps by increasing the radiant flux incident on the base of the cone. This was done by increasing the intensity of a light shining into the chamber through

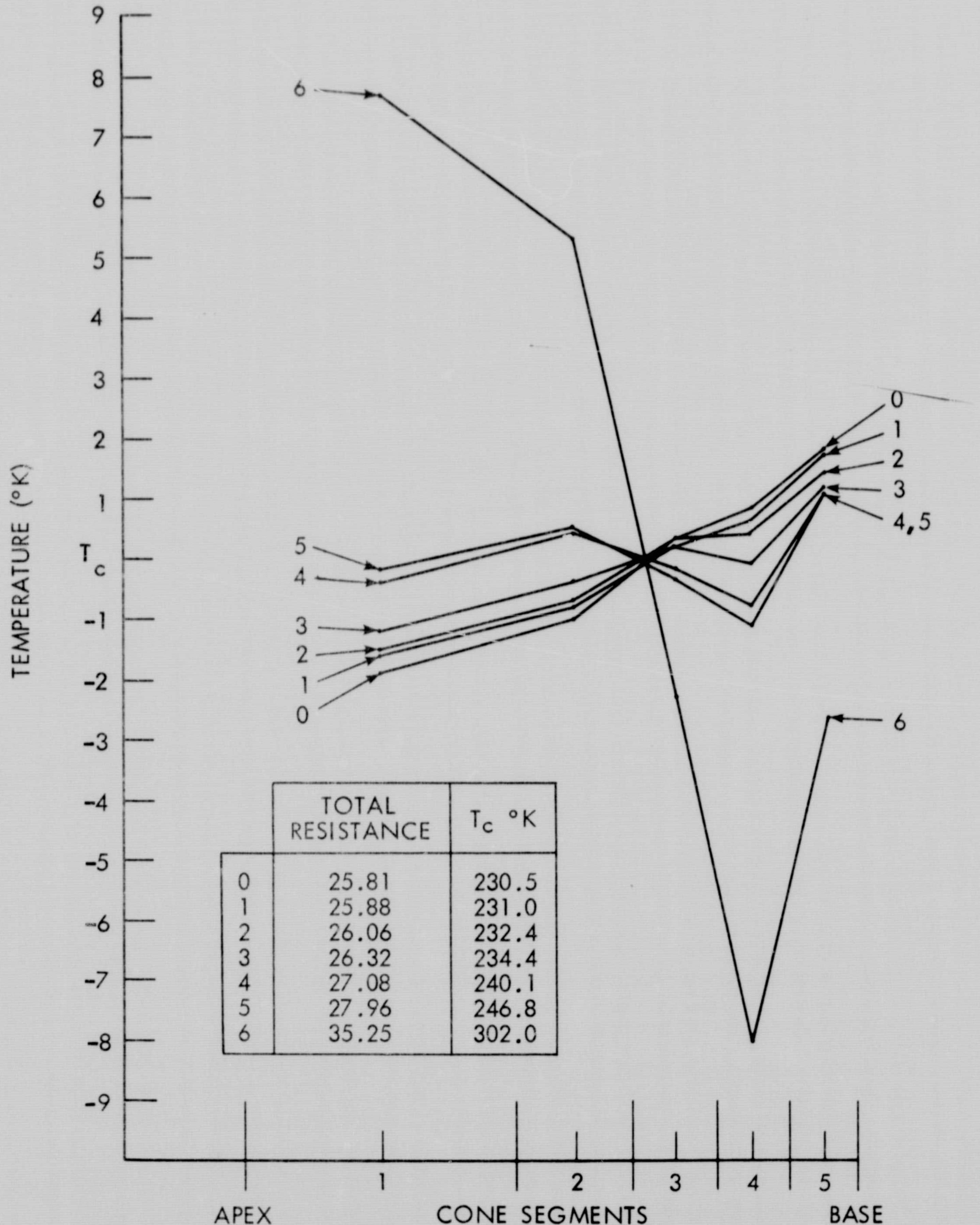


	P_E (mW)	T_c °K
1	0	185.6
2	8.6	188.5
3	19.0	191.0
4	35.4	196.0
5	55.8	202.0
6	82.0	208.1
7	117.0	217.4
8	157.0	226.0
9	268.0	248.2

Figure 8. Temperature Distribution as a Function of Electrical-Power Input

a port with a quartz window. Figure 9 shows the temperature distribution for various temperatures. Again, the clockwise rotation with increasing temperature occurred.

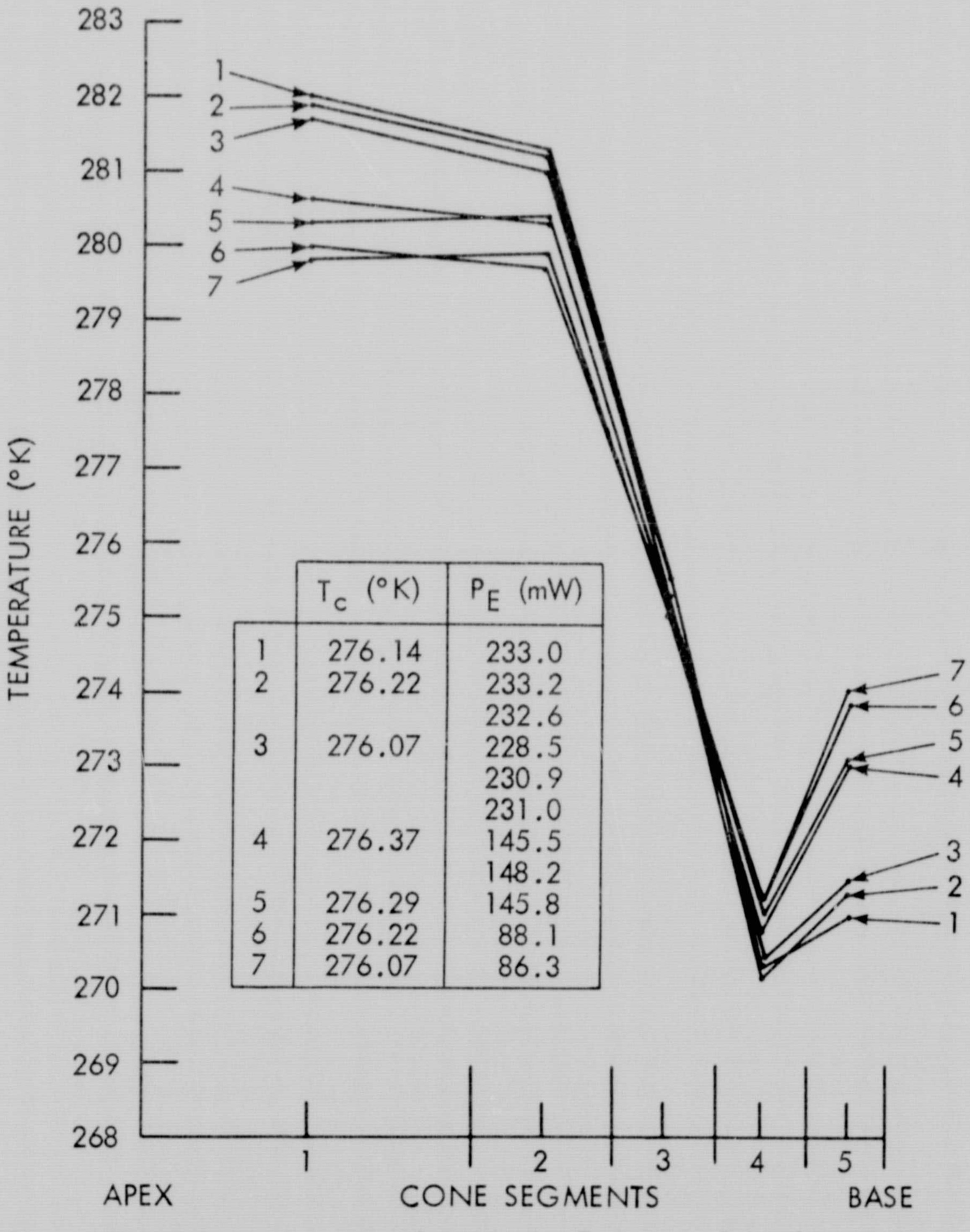
- As both radiant and electrical heating of the cone produced similar results, a fourth test was performed to detect any small differences. This test duplicated what actually happens to the cone in normal operation. The block was at a fixed temperature. The cone was heated electrically to a given temperature (i.e., a given total resistance), and the temperature distribution was measured. Then the cone was illuminated slightly with the light source, and the electrical power decreased the amount



	TOTAL RESISTANCE	T_c °K
0	25.81	230.5
1	25.88	231.0
2	26.06	232.4
3	26.32	234.4
4	27.08	240.1
5	27.96	246.8
6	35.25	302.0

Figure 9. Temperature Distribution as a Function of Incident Radiation

necessary to keep the cone at the same temperature (i.e., the same total resistance). The temperature distribution was again measured. The light intensity was increased and the process repeated. Figures 10 and 11 show the result, a slight counterclockwise rotation about the center of segment three, for two different values of T_C. Decreasing the electrical power to two-thirds its value caused a 1.5°K change in segments 1 and 5. A decrease to one-third its value caused a 2.5°K change in these segments.



	T_c (°K)	P_E (mW)
1	276.14	233.0
2	276.22	233.2
		232.6
3	276.07	228.5
		230.9
		231.0
4	276.37	145.5
		148.2
5	276.29	145.8
6	276.22	88.1
7	276.07	86.3

Figure 10. Temperature Distribution as a Function of Combinations of Electrical and Radiant Inputs (86 to 233 mW Range)

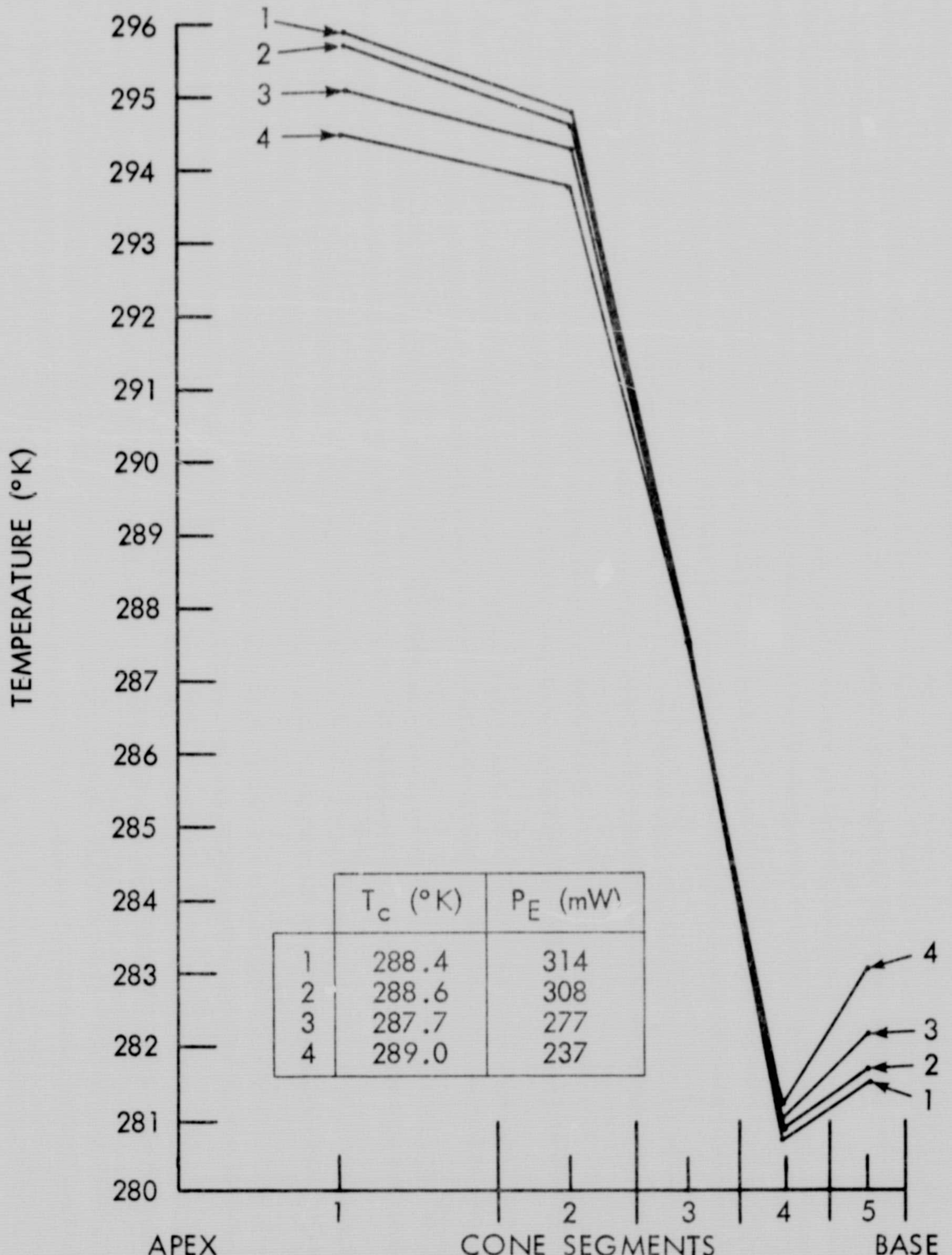


Figure 11. Temperature Distribution as a Function of Combinations of Electrical and Radiant Inputs (237 to 314 mW Range)

Effect of Different Temperature Distributions

As the temperature distributions differed depending on the relative magnitudes of P_E and P_I, another test was run to see if this change in temperature distribution resulted in an error, and, if so, the magnitude of the error. The test consisted of four steps.

- First, the cone was heated electrically to the desired resistance, exposed only to the radiation from the chamber walls. The same current passed through all five segments, as shown in the circuit in Figure 5b. The power required (P_1) and the temperature distribution were measured.

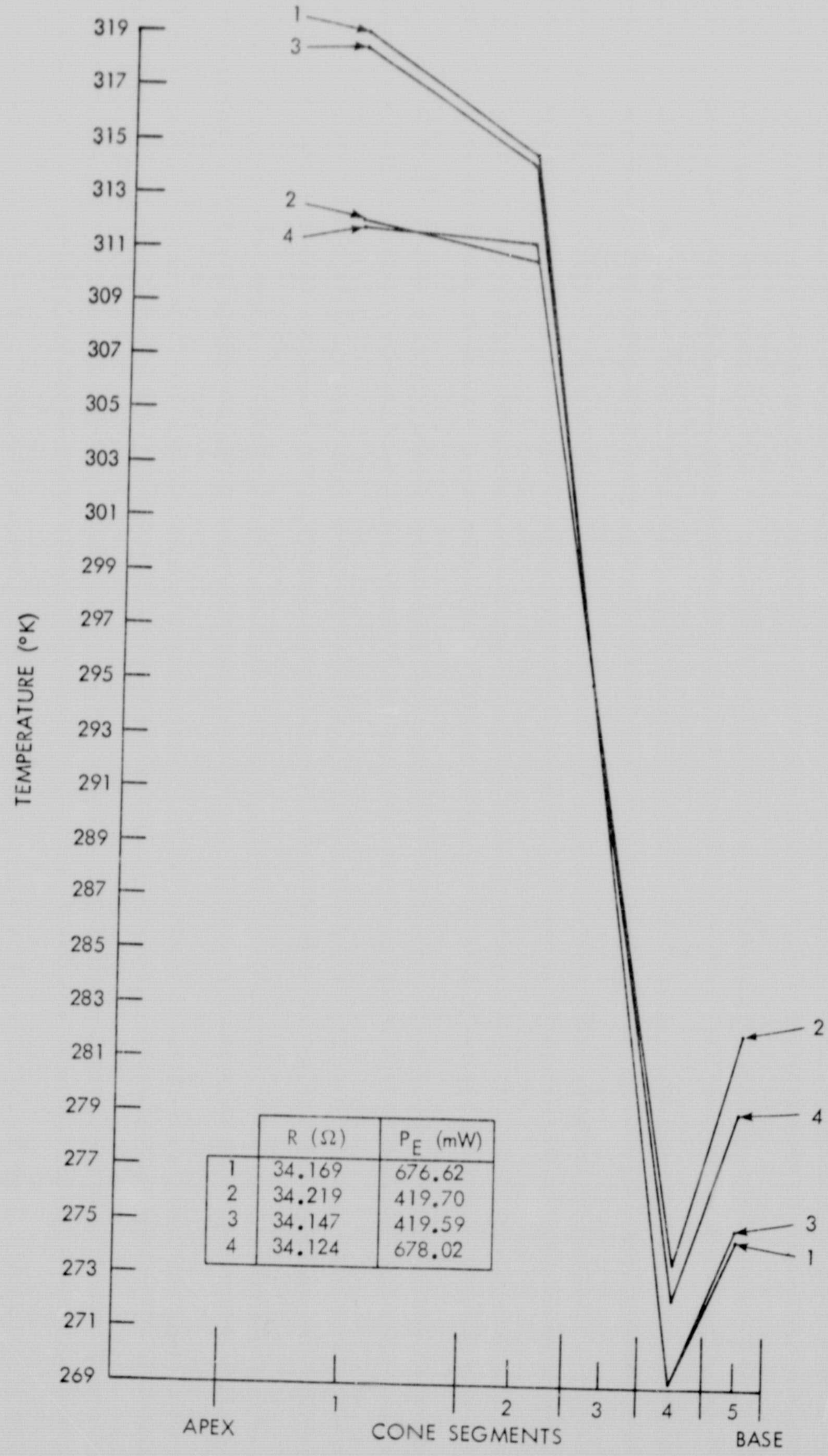
- Second, the cone was exposed to a radiation source (a sun-gun) and maintained at the same desired total resistance. Again the power P_2 and the temperature distribution were measured. P_2 was less than P_1, and the temperature distributions were different.

- Third, the cone segments were reconnected as shown in the circuit in Figure 5c. This permitted each segment to be powered separately and brought to any desired resistance, thus approximating any desired temperature distribution. The cone was exposed to the same radiation-source intensity as in the second step, but the power supplied to each segment was chosen so that the resistance of each segment matched the corresponding resistance in the first step. The power to each segment was measured to get the total power supplied to the cone, P_3.

- Fourth, the cone was exposed only to the radiation from the chamber walls and heated electrically to the desired resistance matching the temperature distribution in the second step. The power to each segment was measured and summed to find the total power, P_4.

As the temperature distribution was the same in steps 1 and 3, the intensity of the source should be equal to $P_1 - P_3$; likewise, the intensity of the source should equal $P_4 - P_2$. Therefore, $P_4 - P_2$ should equal $P_1 - P_3$, and this can be used as a check on the accuracy of the experiment. The real question, though, is: Does $P_1 - P_2 = P_1 - P_3$, as finding $P_1 - P_2$ is the normal way of operating the cone?

The test was run for various combinations of P_E and P_I. With a new block at LN_2 temperature, similar to that used for the solar measurements cone radiometer, whereas earlier temperature distribution tests were run with a heated block at -50° C. Figures 12 and 13 show the results for $P_I = 1/3\, P_{E_0}$ and $P_I = 2/3\, P_{E_0}$. For the first case, $P_I = 1/3\, P_{E_0}$, the temperature-distribution match of step 4 to step 2 was not as good as might be desired. As Figure 12 shows, this is a



	R (Ω)	P_E (mW)
1	34.169	676.62
2	34.219	419.70
3	34.147	419.59
4	34.124	678.02

Figure 12. $P_I = 1/3\, P_{E_o}$

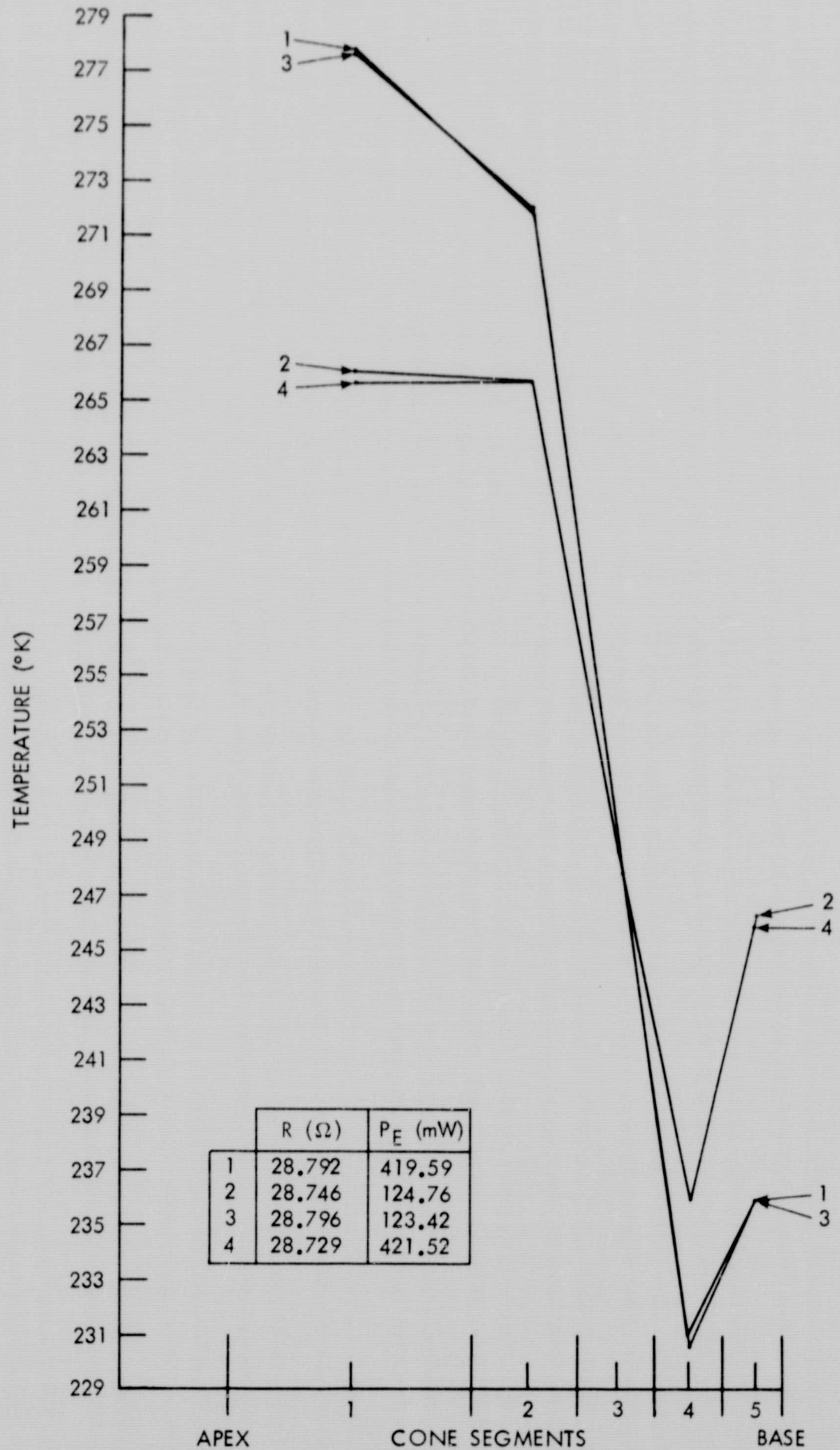


Figure 13. $P_I = 2/3 P_{E_o}$

poor agreement between $P_1 - P_3$ (257.05) and $P_4 - P_2$ (258.32) or a difference of 0.5 percent. Comparing $P_1 - P_2$ (256.92) to the average value of $P_1 - P_3$ and $P_4 - P_2$ (257.69) yields a 0.30-percent difference, whereas a comparison to $P_1 - P_3$ yields only a 0.04-percent difference. For the second case, $P_1 = 2/3 P_{E_0}$, the distributions were more closely matched, as in Figure 13. The agreement between $P_1 - P_3$ (296.17) and $P_4 - P_2$ (296.76) was 0.2 percent. The difference between $P_1 - P_2$ (294.83) and the average of $P_1 - P_3$ and $P_4 - P_2$ (296.47) was 0.56 percent. $P_1 = 2/3 P_{E_0}$ represents about the worst condition under which the cone is operated as far as changes in temperature distribution. Even under these conditions, the error caused by the change in the temperature distribution is less than 0.6 percent, which is less than the uncertainty in the total transmission of the window used with the cone and only slightly larger than the error caused by fluctuations in the temperature of the block.

MATHEMATICAL ANALYSIS

Energy Balance of Cone as a Whole

Calculations were made to see what effect the change in temperature distribution would have on the various powers besides P_E, which was experimentally determined. P_I, the incident power, remained constant independent of any change in the temperature distribution. For the cases examined, P_I consisted of the power emitted by the covered quartz chamber port. P_c, the power emitted from the inner surface of the cone, was calculated using the average temperature of each segment and its form factor. The form factor is the fraction of the power emitted by a segment, a frustum of the cone, which passes through the base of the cone. The assumption was made that each segment was isothermal in order to calculate the form factors and P_c. This assumption, which is not quite correct, greatly simplified the calculations without introducing a significant error.

P_B, the net power exchange between the cone and the surrounding block, was the result of four separate quantities: First, P_{BC}, the power emitted by the block and absorbed the cone, depends only on the block temperature, and thus remains constant. P_{CB}, the power emitted by the outer surface of the cone was calculated using the average temperature of the segments. P_{cu}, the power conducted away from the cone through six copper wires (the electrical leads to the cone) was calculated using the wire diameter, length, block temperature, and segment temperature. The effect of radiative loss from the copper wires was not computed. P_{st} was the power conducted away from the cone through the three steel wires used to support the cone. The steel support wires do not actually touch the cone but are attached by a small spot of epoxy that helps to reduce thermal conductivity. It was impossible to get the exact dimensions of the spot of epoxy, so the dimensions were chosen such that P_{st} would satisfy the equation

$$P_E + P_I + P_{BC} = P_C + P_{CB} + P_{cu} + P_{st}$$

These dimensions were close to the dimensions estimated by looking at the spot of epoxy. The two different temperature distributions, 1 and 4 in Figure 13, were examined in this way. The results are:

Table 5

Effect on Energy Balance of Changes in Temperature Distribution

Power	Case 1 (mW)	Case 4 (mW)	ΔP	% Change	% of Power Supplied or Dissipated
P_E	419.59	421.52	1.93	0.5	97.9
P_I	4.37	4.37	0	0	1.0
P_C	-52.36	-55.93	-3.57	-6.6	-13.0
P_B :					
P_{CB}	-247.41	-240.28	7.13	2.9	-56.0
P_{BC}	4.61	4.61	0	0	1.1
P_{CU}	-69.60	-67.60	2.00	2.9	-15.8
P_{ST}	-62.79	-65.22	-2.43	-3.8	-15.2
Σ	-3.59	1.47	5.06		0

Note that P_E and P_I accounted for 98.9 percent of the power supplied to the cone, and only 1.1 percent came from the block, P_{BC}. Over half the power (56 percent) was emitted from the outer surface of the cone, and only 13 percent was emitted from the inner surface through the base. With a change in temperature distribution, P_C showed the largest percent change in power and P_{CB} showed the largest numerical change. The rest of the power (31 percent) was lost by conduction through the electrical and support wires. The change in temperature caused opposite power changes (+2.9 percent and -3.8 percent respectively) in the electrical and support wires.

Explanation of Temperature Distributions

A model set up to explain the behavior or the temperature distribution of the cone used the following assumptions:

1. $\epsilon = a = 1$ for cone surface

2. each segment isothermal, though different segments may be at different temperatures

3. no conduction losses from the cone

4. no convection losses from the cone

5. $R = R_o (1 + a (T_l - T_o))$ for all T_l

Assumptions 1, 2, and 3, although incorrect, greatly simplify the problem without causing any gross error. Actually, $a \cong \epsilon \cong 0.95$. The segments are not isothermal, but this assumption is necessary in order to calculate the various form factors of the segments. The conductive losses are significant (30 percent) but the effect is only dependent on the first power of temperature, whereas the main effect which is radiative is dependent on the fourth power of temperature. Thus, the conductive losses were omitted. The method was to calculate the energy balance for each segment separately.

In the first case, the only source of power was radiation, assumed to fall uniformly over the cone's inner surface (so that each segment, having the same area, received a fifth of the incident radiation). Each segment radiated energy proportional to its area and to the fourth power of its temperature, from both its inner and outer surface. In addition, each segment absorbed part of the energy it emitted from its own inner surface, and part of the energy emitted from the inner surface of the other segments.

Let $P_k = \epsilon \sigma A_k T_k^4$ be the energy emitted from the inner surface of segment k, F_{ik} the fraction of the energy emitted by segment i which is absorbed by segment k, P_i the energy emitted from the inner surface of segment i, and P the total energy incident on the cone. Then the energy balance equation for segment k should be:

$$\frac{P}{5} + \sum_{i=1}^{5} F_{ik} P_i - 2 P_k = 0$$

Assuming all segments to be at the same temperature, incident power to be 500 mW, and the power entering and leaving the cone as a whole to be the same, the balance for the various segments was not zero, but a net gain existed for segments 1, 2, and 3, and a net loss for 5 and 6. The temperature for the segments with a net gain was raised, and that for the segments with a net loss, was lowered. The temperatures were changed so that the average temperature remained the same, which is the same as keeping the total resistance constant because of assumption 5.

The process was repeated until all the net values were the same (in this case, all slightly a net gain). The reason for the net gain is as follows: If two plates of equal area, A, were at the same temperature T, the total radiation would be

$$A \epsilon \sigma \left(T^4 + T^4\right)$$

If the temperature of one plate were raised t and the temperature of the other plate lowered t (so as to keep their average temperature the same), their total radiation would now be

$$A \epsilon \sigma \quad (T + t)^4 + (T - t)^4 \quad = A \epsilon \sigma \left(2T^4 + 12T^2 t^2 + 2t^4\right)$$

which is greater than $A \epsilon \sigma \ 2T^4$.

Therefore, total power emitted from the surface of the cone increased with the change from a uniform to a nonuniform temperature distribution, and resulted in a net gain for each segment.

Figure 14 shows the resultant calculated temperature distribution, and those for $P = 400$ mW and $P = 200$ mW. Therefore, the observed temperature distributions shown in Figures 7 and 9 are the result of the form factors for the various segments. Segment one reabsorbed more of its own emitted radiation than any other segment (74.12 percent), and segment five reabsorbed the least (17.88 percent). Segment one had the largest total form factor for reabsorbed radiation (0.9508) and segment five had the least (0.4601). Thus, segment one rose in temperature to emit more energy, whereas segment five lowered in temperature to emit less. In Figures 7 and 9, segment four is cooler than segment five, which was not the case in Figure 14, where conductive losses caused a noticeable effect. The three support wires attached to segment four accounted for 15 percent of the power lost by the cone. This relatively good thermal path leads one to expect that segment four would be closer to the block temperature than the other segments.

Next, the source of power to the cone was assumed to be all electrical. As long as all segments were the same temperature (and, thus, exhibited the same

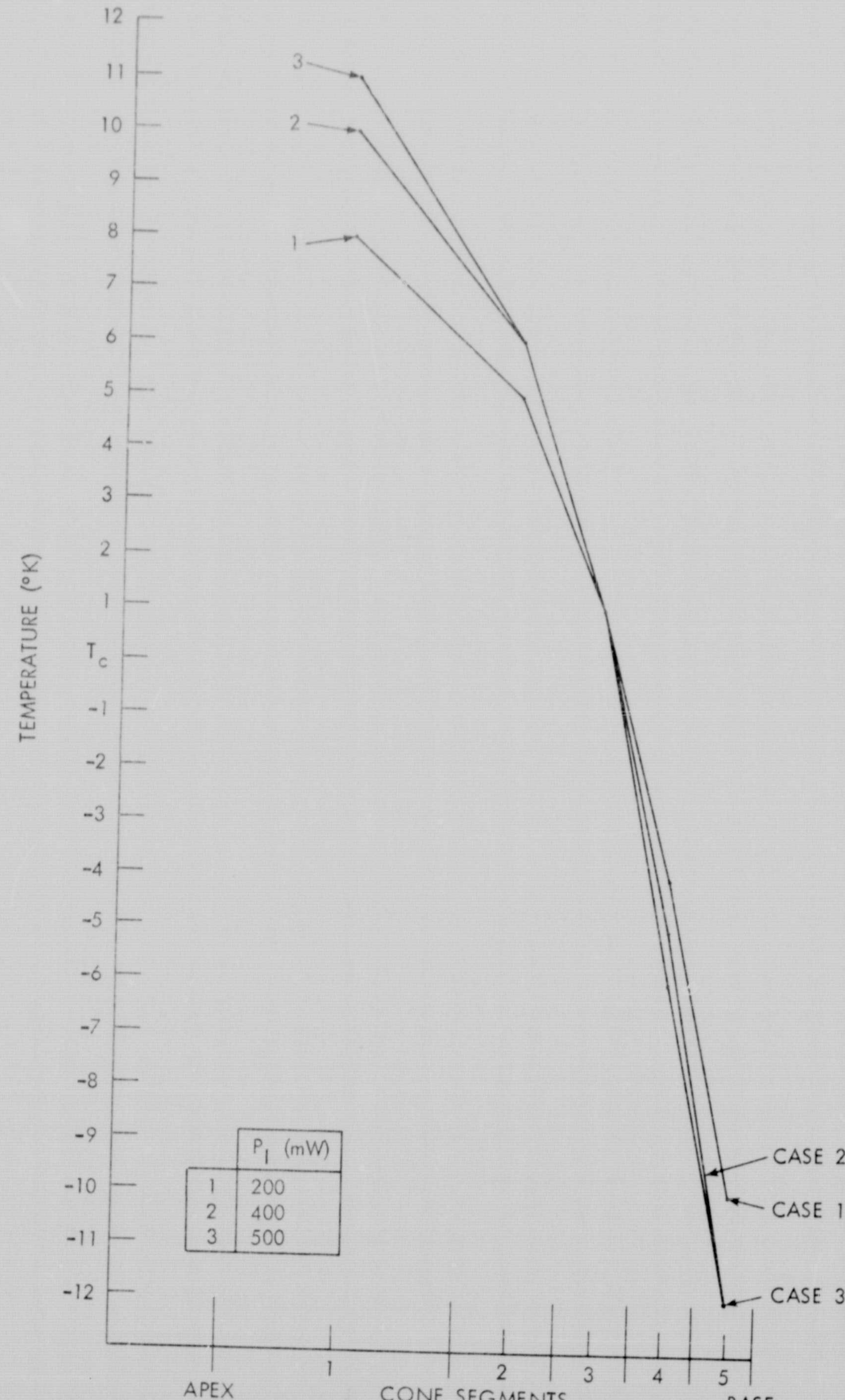


Figure 14. Calculated Temperature Distribution as a Function of Incident Radiation

resistance), the total power was divided evenly among the segments. As the segments were in series, the same current flowed in each. A change in resistance changed the fraction of the total power a given segment received; however, the total power remained constant, because the average temperature (and, thus, total resistance) remained constant. The energy balance equation then became

$$\dot{P} = I^2R = I^2\sum_{i=1}^{5} R_i$$

$$I^2R_k + \sum_{i=1}^{5} F_{ik}\ P_i - 2P_k = 0$$

The same process of changing temperatures was repeated. Figure 15 shows the results, which are similar to the observed distributions in Figure 8. Again, the observed results were due to the form factors; however, the fact that the incident power to a given segment increased as its temperature increased caused the distribution to change even more. This second effect, though, was dependent only on the first power of the temperature. This effect is what causes the different distributions, depending on the relative magnitudes of the electrical power and the incident radiation, as shown in Figures 10 and 11. Figures 16 and 17 show the corresponding calculated distributions.

The basic cause of the temperature distribution is the radiative properties of the cone shape. The difference between incident radiation and electrically caused distributions, which is slight, is due to the nonuniform resistance of the wire caused by its nonuniform temperature.

OPERATIONAL PROBLEMS

Measuring Absorptive Area

Some questions about the cone are matters of operation rather than of basic validity. Mentioned earlier was the task of determining $A\alpha$, the absorptive area of the cone; this depends on a measurement of the base area of the cone, the value of α for the paint, and the absorption-enhancing properties of the cone-shaped cavity.

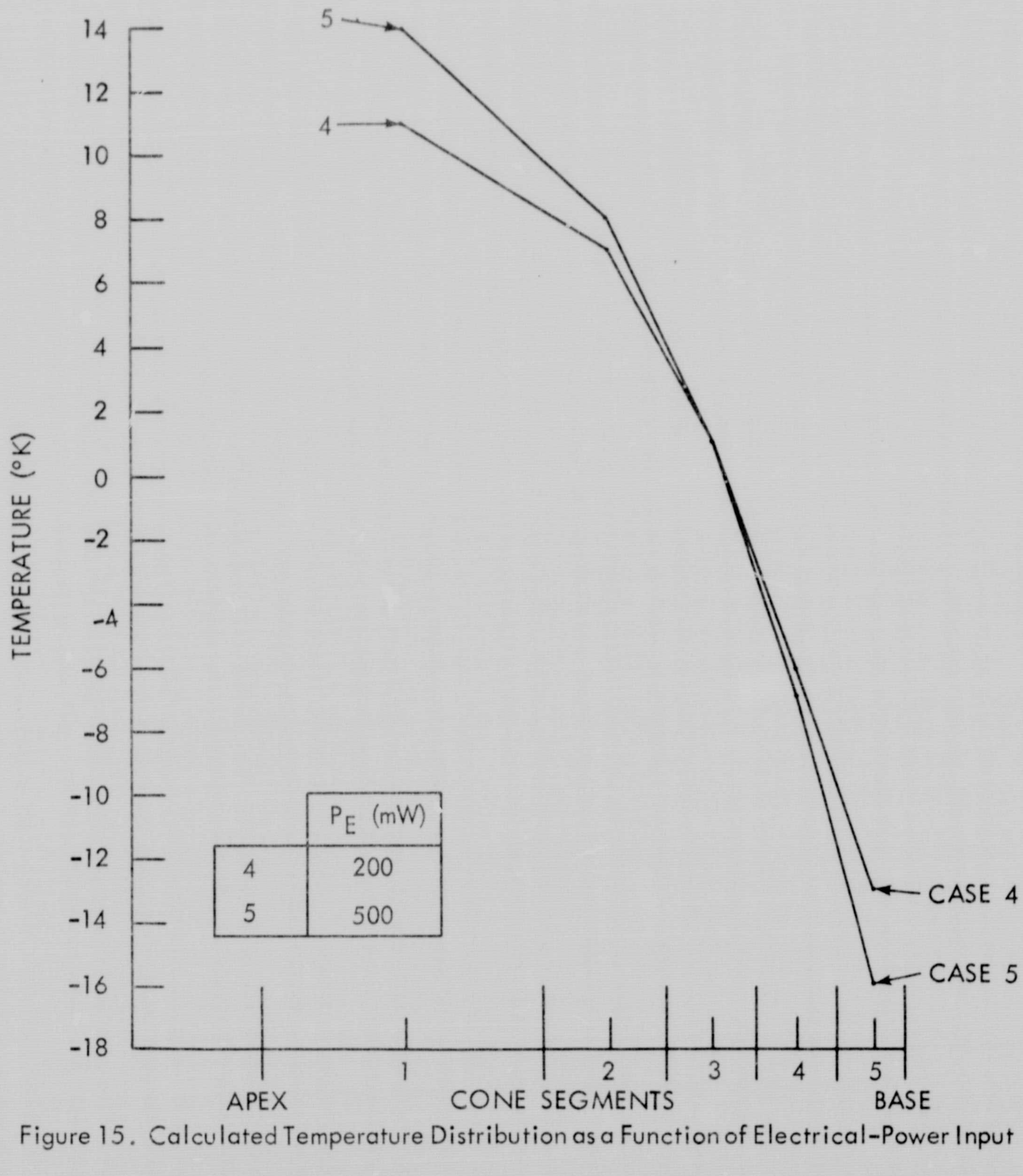


Figure 15. Calculated Temperature Distribution as a Function of Electrical-Power Input

Determining Zero Reading

Another difficulty is determining a value for P_{E_0}. This requires a zero value for P_I, the incident radiation, which is difficult to achieve. A possible solution is to minimize P_I by measuring the radiation from a cold black plate, which could be calculated if its value were not negligible. If a window (such as sapphire) is placed in front of the radiometer, any surface below 300° K will appear

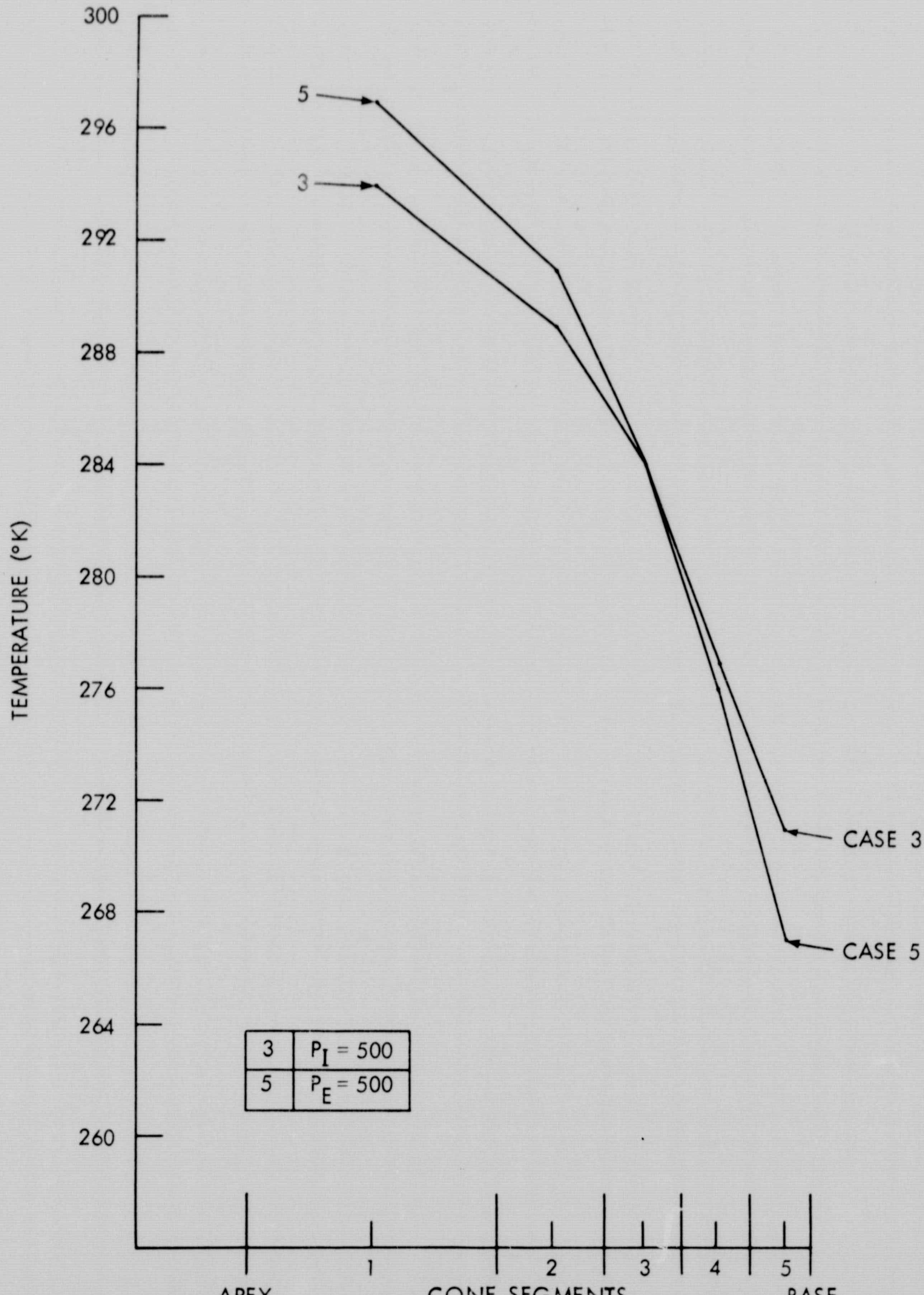


Figure 16. Calculated Temperature Distribution Difference Depending on Relative Amounts of Electrical or Radiant Input, Cases 3 and 5

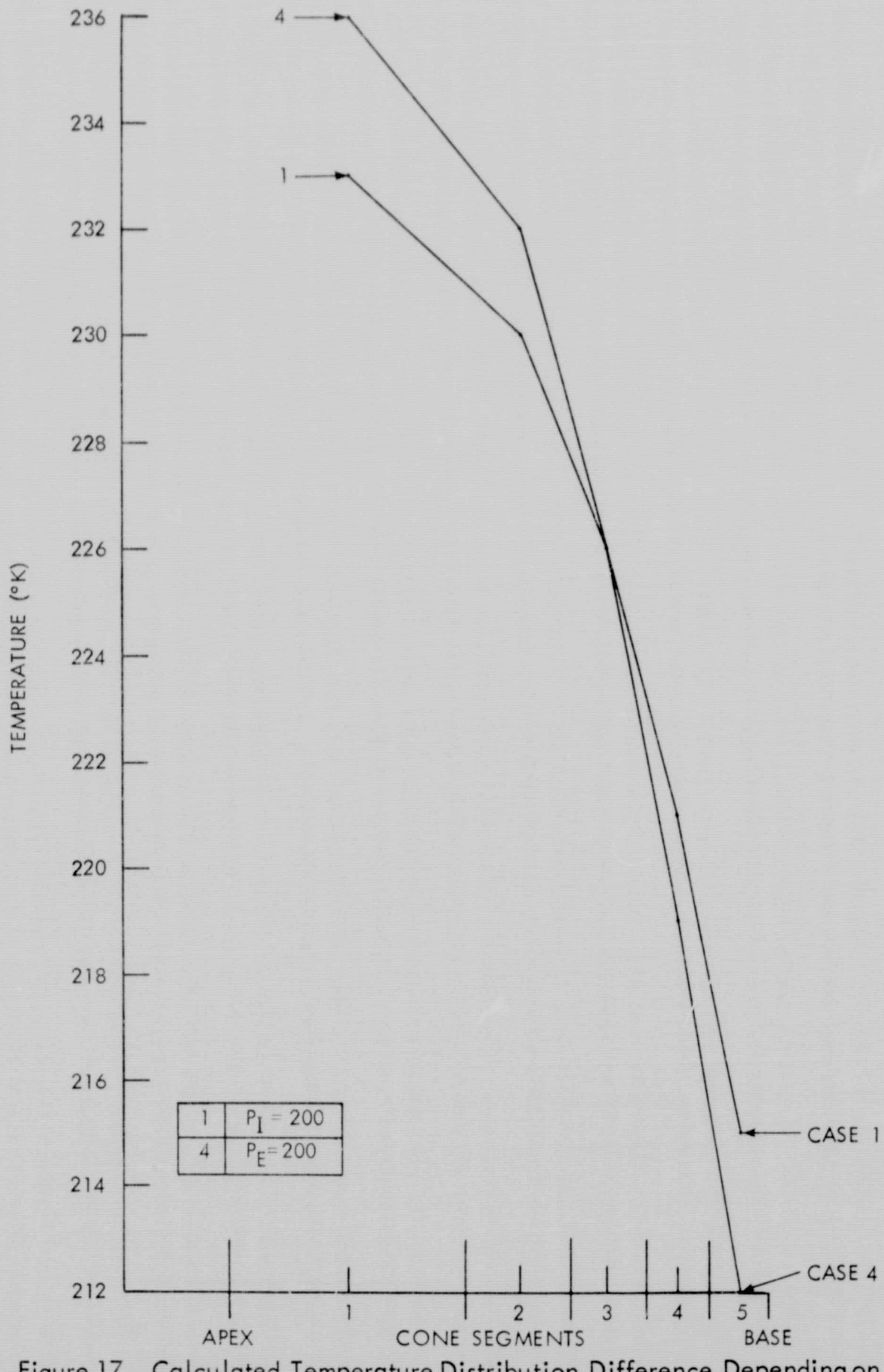


Figure 17. Calculated Temperature Distribution Difference Depending on Relative Amounts of Electrical or Radiant Input, Cases 1 and 4

to the radiometer to be emitting no radiation, because of the long-wavelength cutoff of sapphire; use of a window, however, introduces spectral sensitivity problems. In measuring changes in radiant intensity level, there is no need to determine P_{E_0} for $P_I = 0$, as any change in P_I will cause an equal change in P_E.

Choice of Block and Cone Temperatures

Another operational problem is the choice of block temperature and cone temperature. The size and the total resistance of the wire forming the cone put a maximum value on P_E, which is also governed by the maximum power output of the automatic control system for the cone. Thus, the choice of a temperature at which the block is to be maintained immediately establishes a maximum value for the temperature of the cone. The precision with which the block and cone temperatures are maintained determines the extent to which their net energy exchange, P_B, remains constant. With these restrictions in mind, the cone-temperature tolerances were computed for three different block temperatures, keeping P_B about the same in all three cases. The tolerances are expressed as the number of degrees of change necessary to cause a 0.25-percent change in P_B. The tolerance in cone temperature is also expressed for a nickel-wire cone in terms of percentage of change in the cone's resistance.

Table 6

Low Temperature Tolerances as a Function of Block Temperature

T Block (°K)	T Cone (°K)	Δ T Block (°K)	Δ T Cone (°K)	Change in R (%)
100	300	±5.0	±0.2	0.12
200	320	±1.0	±0.2	0.12
300	360	±0.2	±0.1	0.06

CONCLUSION

The basic principle underlying the theory of the cone is the equivalent substitution of electrical power for absorbed radiative power. In actual operation of the cone, this substitution is not exactly equivalent because a change occurs in temperature distribution. A test showed that different temperature distributions were not equivalent, and did not require equivalent amounts of electrical power to establish. Thus, the cone, like many instruments, does not perform in reality as its ideal case does. However the error due to the change in temperature distribution was found to be small, and of the order of magnitude of the other uncertainties presently associated with the cone. These uncertainties are:

Change in temperature distribution	-0.60% max (for $P_I = 2/3\, P_E$)
Fluctuation in block temperature	±0.30%
Uncertainty in value of A	±0.25%
Uncertainty in value of a	±0.35%
	-0.60% ±0.30% rms

Therefore, the absolute accuracy of the cone can be safely placed at ±1 percent or better even without correcting for the temperature-distribution change.